Nonholonomic mechanics for perfect fluids

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Perfect fluid models, such as the incompressible Euler equations, possess a remarkable geometric structure which can be described using the classical Lagrangian and Hamiltonian formalism. This structure is responsible for the numerous properties of the solutions, including conservation of energy and circulation, which directly descend from the symmetries of the system. Although powerful numerical techniques have been derived to tackle this class of problems, two main difficulties arise when dealing with fluids. Firstly, the infinite-dimensional nature of the problem requires a truncation that would preserve its variational nature. Secondly, numerical integrators should be designed in the Eulerian setting in order to avoid complications related to moving or deforming meshes. [1] addressed both issues by approximating the Lie algebra of divergence-free vector fields by its action on piecewise constant functions defined on a triangulation of the computational domain and imposing a nonoholonomic constraint to obtain a sparse matrix algorithm. In this talk, we build on their results to develop a high order finite element geometric formulation of perfect incompressible fluids. The main idea of the method is to reinterpret the spatial discretization of the velocity fields as a nonholonomic constraint, and to define an isomorphism with a suitable Lie algebra. We present some preliminary results obtained with this formulation and we set the scene for further developments of the algorithm.

References

[1] Pavlov, D. and Mullen, P. and Tong, Y. and Kanso, E. and Marsden, J. E. and Desbrun, M. (2011) Structure-preserving discretization of incompressible fluids. Physica D, 240 (6). pp. 443-458. ISSN 0167-2789.