New answers to an old question: Essential underlying ODE versus inherent explicit ODE

Roswitha März (Humboldt-Universität zu Berlin)

In the context of differential-algebraic equations (DAEs) one finds different associated explicit ordinary differential equations (ODEs): completion ODEs, underlying ODEs, essential underlying ODEs and inherent ODEs. Each of them is occasionally considered to control the flow of the DAE. However, how are they related to each other? This question has been asked long time ago, also in early NUMDIFF-conferences, but it has been partly answered only.
We will put straight the notions and then take a closer look to essential underlying ODEs (EUODEs) and inherent explicit regular ODEs (IERODEs). The latter arise within the projector based decoupling of DAEs. The former has been introduced by U. Ascher and L.R. Petzold for Hessenberg-form index-2 DAEs in 1992 by means of special transformations. We extend this notion to general arbitrary-index linear DAEs, no matter in which form, without or with properly involved derivative.
Any regular linear differential-algebraic equation (DAE) features a unique IERODE living in the given space. In contrast, there are several EUODES living in a transformed space with possible minimal dimension. In 2005 it has been pointed out by K. Balla and Vu Hoang Linh that, for index-2 Hessenberg-form DAEs, an EUODE is a condensed IERODE. We show that this applies also in general.
The understanding of the relation between the IERODE and the EUODEs enables us to uncover the stability behavior of the DAE flow. We discuss several questions concerning Lyapunov stability, Lyapunov spectra, Lyapunov regularity, and Perron identity.