Geometric Integration of Degenerate Lagrangian Systems

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The talk aims at answering the question of how to derive long-time stable numerical methods for the integration of particle trajectories in fusion plasmas?

In numerical simulations of dynamical systems it is crucial to preserve certain structures of the equations in order to obtain accurate results, especially - but not only - in long-time simulations. Most systems encountered in plasma physics are Hamiltonian and therefore have a rich geometric structure, most importantly symplecticity and conservation of momentum maps. As most of these systems are formulated in noncanonical coordinates, they are not amenable to standard symplectic methods which are popular for the integration of canonical Hamiltonian systems. An alternative route towards the systematic derivation of structure-preserving numerical methods is provided by the Lagrangian frame. The discretisation of the variational formulation leads to so called variational integrators which can be seen as the Lagrangian equivalent to symplectic methods. However, for noncanonical Hamiltonian systems most often the Lagrangian is degenerate. This degeneracy gives rise to instabilities of the variational integrators which need to be overcome in order to make long-time simulations possible.

We will review basic ideas of geometric or structure-preserving numerical integration and introduce the method of variational integrators. The guiding-centre system, a particularly important system from plasma physics, will be used to exemplify the problems which arise for noncanonical Hamiltonian systems with degenerate Lagrangians. We will show how the resulting instabilities of the variational integrators can be eliminated by appropriate projection methods in order to obtain truly long-time stable numerical methods.