Efficient time integration of the Maxwell-Klein-Gordon equation in the non-relativistic limit regime

Patrick Krämer (KIT Karlsruhe), Katharina Schratz

Solving the Maxwell-Klein-Gordon (MKG) equation in the non-relativistic limit regime is numerically very delicate as the solution becomes highly oscillatory in time. In order to resolve the oscillations, standard integration schemes require severe time step restrictions.
The idea to overcome this numerical challenge lies in the asymptotic expansion of the solutions, which allows us to filter out the high frequencies explicitly (see [1] for the case of the Klein-Gordon equation).
More precisely, this ansatz allows us to break down the numerical task to solving a non-oscillatory Schrödinger-Poisson system (SP), which can be carried out very efficiently without any additional time step restriction for example by applying splitting methods (cf. [2]). This formally derived non-relativistic limit of the MKG equation has already been studied from an analytical point of view in [3].
In my talk I want to present the ideas of the convergence proof for the MKG equation to the SP system and give some numerical results.

References

[1] E. Faou and K. Schratz, Asymptotic preserving schemes for the Klein-Gordon equation in the non-relativistic limit regime, *Numer. Math., Vol. 126, 2014, No. 3*, pp. 441–469.

[2] C. Lubich, On splitting methods for Schrödinger-Poisson and cubic nonlinear Schrödinger equations, *Math. Comp., Vol.77, 2008, No.264*, pp. 2141–2153.

[3] N. Masmoudi and K. Nakanishi, Nonrelativistic limit from Maxwell-Klein-Gordon and Maxwell-Dirac to Poisson-Schrödinger, *Int. Math. Res. Not., 2003, No. 13*, pp. 697–734.