Newmark type time integration methods for strongly damped mechanical systems

Markus Köbis (Institute of Mathematics, Martin Luther University Halle-Wittenberg), Martin Arnold

The simulation of mechanical or biological structures often results in systems of ODEs with very strong dissipative forces. In this talk we will adress the prototypical equations

$$M(q^\delta)\ddot{q}^\delta = f(t, q^\delta, \dot{q}^\delta) - \frac{1}{\delta} G^\top(q^\delta) G(q^\delta) \dot{q}^\delta, \quad (0 < \delta \ll 1)$$

for a given vector-valued function $f\colon \mathbb{R} \times \mathbb{R}^{n_q} \times \mathbb{R}^{n_q} \to \mathbb{R}^{n_q}$, and matrix-valued functions $M\colon \mathbb{R}^{n_q} \to \mathbb{R}^{n_q \times n_q}$ (the symmetric positive definite mass matrix) as well as $G\colon \mathbb{R}^{n_q} \to \mathbb{R}^{n_g \times n_q}$, which defines an invariant manifold $\mathfrak{M} := \{(q, v) : G(q)v = 0\}$ and is supposed to have a full-rank for all arguments $q \in \mathbb{R}^{n_q}$. From a DAE viewpoint this system may be seen as a singular perturbation of the index two system

$$\begin{aligned} M(q^0)\ddot{q}^0 &= f(t, q^0, \dot{q}^0) - G^\top(q^0)\lambda^0, \\ 0 &= G(q^0)\dot{q}^0 \end{aligned}$$

well known in the context of multibody system dynamics. We will apply time integration methods of Newmark type to the above equations and investigate their convergence properties by extending results from the DAE case to this singularly perturbed setting. Numerical results for benchmarks of small and moderate size will be presented and confirm the theoretical findings.