Generalized ROW-Type Methods for Solving Semi-Explicit DAEs of Index-1

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Since being introduced in the sixties and seventies, semi-implicit Rosenbrock-Wanner (ROW) methods have become an important tool for the time-integration of ODE and DAE problems. Over the years, these methods have been further developed in order to save computational effort by regarding approximations with respect to the given Jacobian [5], reduce effects of order reduction by introducing additional conditions [2, 4] or use advantages of partial explicit integration by considering underlying Runge-Kutta formulations [1]. As a consequence, there is a large number of different ROW-type schemes with characteristic properties for solving various problem formulations given in literature today.
In order to combine the most relevant ROW-type representations, a new class of methods for computing semi-explicit DAE problems of index-1 is introduced. The generalized formulation covers many well-known ROW-type methods including schemes for problems with additive and partitioned splitting into stiff and non-stiff components. Thus, it enables to take the form of the most advantageous integration strategy with respect to characteristics of a given problem. Satisfying corresponding order conditions, all methods involved can be realized with just one single set of coefficients regarding appropriate approximations with respect to given Jacobian entries. In this context, new order conditions for using non-exact Jacobians when applying ROW methods to DAE problems are introduced by extending the theory of Roche [3].

References

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