Convergence of ADI schemes for two-dimensional convection-diffusion equations with mixed derivative term

Karel in 't Hout (University of Antwerp), Maarten Wyns

Alternating Direction Implicit (ADI) schemes are well-known in the numerical solution of multidimensional time-dependent partial differential equations (PDEs) arising in financial mathematics. The Craig-Sneyd (CS), Modified Craig-Sneyd (MCS) and Hundsdorfer-Verwer (HV) schemes form three popular ADI schemes. A structural analysis of their fundamental properties, notably convergence, is of main interest. Up to now, however, a rigorous convergence result has only been derived in the literature for the HV scheme and only in the case of one-dimensional PDEs. In this talk we shall present new results revealing that, under natural stability and smoothness conditions, the CS, MCS and HV schemes all possess a temporal order of convergence equal to two, uniformly in the spatial mesh width, whenever they are applied to two-dimensional convection-diffusion equations with mixed derivative term. The obtained convergence results are illustrated by numerical experiments for contemporary stochastic volatility models.