Convergence analyses of the Peaceman–Rachford and Douglas–Rachford Schemes for Semilinear Evolution Equations

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We analyse convergence of the Peaceman–Rachford and Douglas–Rachford time discretization schemes for semilinear evolution equations. The vector field of the equation is assumed to be the sum of two unbounded dissipative operators. A setting in which the Peaceman–Rachford and Douglas–Rachford splitting methods exhibit excellent stability properties.
Convergence orders are given when the vector field is the sum of a linear and a nonlinear operator. The full range of possible orders (including convergence without order) are given depending on the regularity of the solution. In contrast to previous convergence order studies we do not assume Lipschitz continuity of the nonlinear operator.
Taking the analysis further we combine these temporal discretizations with convergent spatial discretizations. We do this in the setting of linear evolution equations and prove optimal, simultaneous, space-time convergence orders. We observe how the results profit from the excellent local error structures of the Peaceman–Rachford and Douglas–Rachford schemes.
Applications include semilinear reaction-diffusion equations. The convergence orders are illustrated by numerical experiments.

Related articles:
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SIAM J. Numer. Anal., 51(4):1900–1910, 2013.
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A full space-time convergence order analysis of operator splittings for linear dissipative evolution equations.
Preprint, Lund University, `http://www.maths.lth.se/na/staff/erik/`, 2014.