Improvement of dimension splitting methods

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A *dimension splitting method* may suffer from a severe order reduction when applied to an inhomogeneous evolution equation of the form

$$u'(t) = \mathcal{L}u(t) + g(t), \quad u(0) = u_0$$

on $L^2(\Omega)$, where $\Omega = (0,1)^2$ and $\mathcal{L} = \partial_x(a\partial_x) + \partial_y(b\partial_y)$ is an uniformly strongly elliptic operator with $a, b \in \mathcal{C}^2(\overline{\Omega})$ and $D(\mathcal{L}) = H^2(\Omega) \cap H_0^1(\Omega)$. For instance, the *exponential Strang splitting* involving the split operators $\mathcal{A} = \partial_x(a\partial_x)$ and $\mathcal{B} = \partial_y(b\partial_y)$ converges, in general, with order $5/4 - \varepsilon$ in time for arbitrarily small $\varepsilon > 0$ due to arising corner singularities in the derivatives of the solution. However, one might modify the problem such that the full convergence order of 2 is achieved.
In this talk, regularity results for the corresponding stationary problem on $(0,1)^d$ with $d > 2$ are presented and their consequences for the improvement of dimension splitting methods in higher dimensions are discussed.