High-Order Explicit Local Time-Stepping Methods For Wave Propagation

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In the presence of complex geometry, adaptivity and mesh refinement are certainly key for the efficient numerical simulation of wave phenomena. Locally refined meshes, however, impose severe stability constraints on any explicit time-marching scheme, where the maximal time-step allowed by the CFL condition is dictated by the smallest elements in the mesh. When mesh refinement is restricted to a small subregion, the use of implicit methods, or a very small time-step in the entire computational domain, are very high a price to pay.
Explicit local time-stepping schemes (LTS) overcome the bottleneck due to a few small elements by using smaller time-steps precisely where the smallest elements in the mesh are located. For wave equations without damping, leap-frog based LTS methods lead to high-order explicit LTS schemes, which also conserve the energy. For damped wave equations, Adams-Bashforth or Runge-Kutta based LTS methods lead to efficient LTS schemes of arbitrarily high accuracy. When combined with a finite element discretization in space with an essentially diagonal mass matrix, the resulting time-marching schemes are fully explicit and thus inherently parallel. Numerical experiments with continuous and discontinuous Galerkin finite element discretizations validate the theory and illustrate the usefulness of these local time-stepping methods.