Enforcing power-law kinetic energy spectra under conservative discretizations of fluids

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As noted by Ascher and McLachlan, inviscid fluid fluid dynamics presents a challenge to geometric integration. Firstly, although continuum formulations of inviscid fluid dynamics possess a well-known Poisson structure, it has proven particularly challenging to preserve such structure under discretization. Secondly, although the viscous length scale encountered in practical applications such as large scale atmosphere and ocean flows is well below the grid scale, justifying an inviscid assumption, the effect of viscous dissipation on these scales implies a power law distribution of kinetic energy in the presence of large scale forcing. Such a kinetic energy distribution will not be observed when energy and enstrophy preserving schemes are used. Consequently, in practice some form of artificial viscosity is typically employed. In this talk we describe a method for preserving an observed steady state kinetic energy spectrum by applying Nosé-Hoover thermostats to energy shells in Fourier space. The new technique is computationally cheap and leads to only weak perturbation of large scale dynamics, point-wise autocorrelation functions, and ensemble dispersion.