Geometric methods for differential equations in applications of computer animation

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Motion of virtual characters in video games is usually represented using a skeletal animation. The underlying skeleton consists of bones connected by joints. Animation of virtual characters relies on collections of data obtained by recording the movements of actors. The data consists of curves tracking the positions of the bones throughout the motion, and these curves can be processed by mathematical methods to produce new motions. In practice, the data consists of curves in $SO(3)^N$, where N is the number of bones in the skeleton. In the existing literature these curves are represented using Euler angles and neglecting the underlying Lie group structure, [7], [2]. We here report on the results we obtained by appropriately including the underlying geometric structure in the mathematical models and their numerical discretizations. The intrinsic geometric formulation is robust and works very well in problems of motion blending and curve closing, where earlier the same performance could be obtained only by using ad hoc strategies, e.g. keeping track of carefully chosen feature points along the curves.
We will briefly discuss how techniques from shape analysis on Lie groups can be successfully applied to computer animation treating character animations as points in an infinite dimensional manifold. This manifold is in fact an infinite dimensional Lie group where we are interested in computing distances and geodesics. As expected by smimilar experience in the case of shapes on Euclidean spaces [1], the choice of metrics on these infinite dimensional manifolds are a trade off between ease of computation and the needs dictated by the application at hand. In particular we will discuss elastic metrics and their relation to the L_2 norms for curves on the Lie algebra. Furthermore the extension to higher order Sobolev type norms will be considered.
Working with curves of animation data, we are interested in particular in two mathematical problems: computing geodesics on the infinite dimensional Lie group; and projecting open curves onto the submanifold of closed curves. We will discuss the modeling of these problems by designing appropriate gradient flows. We will finally address the numerical discretization of these PDEs and show the advantages of the proposed approach in animation of character motion.

References

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