The construction of high order G-symplectic methods

John Butcher (University of Auckland)

A general linear method

$$\begin{bmatrix} A & U \\ B & V \end{bmatrix}$$

is "G-symplectic" if for some non-singular symmetric matrix G and diagonal matrix D,

$$\begin{bmatrix} DA + A^{\mathsf{T}}D - B^{\mathsf{T}}GB & DU - B^{\mathsf{T}}GV \\ U^{\mathsf{T}}D - V^{\mathsf{T}}GB & G - V^{\mathsf{T}}GV \end{bmatrix} = 0.$$

Many examples of these methods are known and their behaviour is now well understood, both theoretically and experimentally. The focus is now on deriving high order methods in the anticipation that they will provide accurate and efficient integration schemes for mechanical and other problems. One of the starting points is an analysis of the order conditions (Butcher, J., Imran G.,Order conditions for G-symplectic methods, BIT, DOI 10.1007/s10543-014-0541-x (2015.)). It was shown that the order conditions are related to unrooted trees in a similar way to what is known for symplectic Runge–Kutta methods (Sanz-Serna J. M., Abia L., Order conditions for canonical Runge-Kutta schemes, SIAM J. Numer. Anal. 28, 1081–1096 (1991)). Starting from the order conditions, simplifications can be made by assuming time-reversal symmetry and enhanced stage order. Even after high order methods have been found, the construction of suitable starting schemes is an essential step before working algorithms can be built.