Implicit–explicit general linear methods with inherent Runge–Kutta stability

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We consider the initial value problem for the system of ordinary differential equations of the form

$$\begin{cases} y'(t) &= f(y(t)) + g(y(t)), \quad t \in [t_0, T], \\ y(t_0) &= y_0 \in \mathbb{R}^m, \end{cases}$$

where $f\colon \mathbb{R}^m \to \mathbb{R}^m$ represents non-stiff part and $g\colon \mathbb{R}^m \to \mathbb{R}^m$ stiff part of the system.
For efficient integration of such systems we consider the class of implicit-explicit (IMEX) methods, where the non-stiff part $f(y)$ is integrated by explicit method, and stiff part $g(y)$ is integrated by implicit method.
In this talk we present extrapolation based approach to construct IMEX general linear methods. We begin with implicit general linear methods with inherent Runge–Kutta stability (IRKS) of order p, stage order q, with s internal stages and r external approximations. Here, we assume that $p = q = s - 1 = r - 1$. Next, we extrapolate the implicit values in non-stiff terms using available quantities from previus step and current step. The dimensions of coefficient matrices and degree of stability polynomial are doubled.
Our aim is to find IMEX methods with large regions of absolute stability of explicit part assuming that implicit part of the method is A- or L-stable. It is done by numerical search in the space of free parameters of the method and free parameters of extrapolation. We provide examples of such methods that have larger regions of absolute stability than in similar classes of general linear methods. Numerical examples are also given which illustrate good performance of these schemes.