Analytic and numerical solutions to the problem of double pendulum stabilization

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The paper is about one passive control problem. This is the problem of controlling plane motions of two-mass parametric pendulum in a uniform gravitational field. The pendulum is modeled by two equivalent weightless rods with two equivalent point masses moving along a circle. Our control parameter is the angle between the rods of the pendulum. So it is a function depending on the representative point of the gravity center of the pendulum in the phase plane. In this paper, we construct two control laws for excitation and damping the pendulum near the lower equilibrium position by means of the swing principle. The formulated problem is solved then by the method of Lyapunov's functions of the classical stability theory, and this solution is derived in the class of continuous functions in closed form. As an application of our result, we consider the problem of gravitational stabilization of two diametrically opposite relative equilibrium positions of a satellite in a circular orbit with in-plane perturbations. The theoretical outcome of this research is confirmed by numerical simulations in MATHEMATICA and in agreement with practical experimentation.