

A flow-on-manifold formulation of DAEs. Application to positive systems.

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We generalize the concept of the flow from ordinary differential equations (ODEs) to differential-algebraic equations (DAEs). Using the framework of derivatives arrays and the strangeness-index, we identify those DAEs that are uniquely solvable on a particular set of initial values and define a flow, the mapping that uniquely relates a given initial value with the solution through this point. The flow allows to study system properties like invariant sets, stability, monotonicity or positivity. For DAEs, the flow further provides insights into the manifold onto which the system is bound to and into the dynamics on this manifold. Using a projection approach to decouple the differential and algebraic components, we give an explicit representation of the flow that is stated in the original coordinate space. This concept allows to study DAEs whose dynamics are restricted to special subsets in the variable space, like a cone or the nonnegative orthant. We give a uniform description of flow invariance for constrained and unconstrained systems and specialize this result for positive systems, i.e., systems leaving the nonnegative orthant invariant. Positive systems arise in every application, in which the analyzed values represent real matter, like the amount of goods, individuals or the density of a chemical or biological species. Simulating these processes, the loss of positivity leads to severe stepsize restrictions or failures in the simulation. Having conditions on the flow, when the system is positive provides the basis to study positive preserving discretization for DAEs.