Variance reduction techniques for the numerical simulation of the stochastic heat equation

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We consider the finite dimensional stochastic heat equation (obtained by spatial discretisation)

$$\begin{aligned} du(t) &= \Delta_{h,0} u(t) dt + \sigma u(t) dW(t) \\ u(0) &= u_0, \end{aligned}$$

where $\Delta_{h,0}$ denotes the discrete Laplacian with homogeneous Dirichlet boundary conditions and $W(t)$ is a Q-Wiener process. For growing diffusion parameter $\sigma \in \mathbb{R}^+$ the equilibrium solution of the system eventually gets mean-square unstable, however it takes an unreasonably large number of numerical trajectories to see this instability in Monte-Carlo simulation. We will discuss the practicability and the influence of variance reduction techniques, namely importance sampling via Girsanov's theorem and control variates, on the Monte-Carlo estimation. This talk is based on joint work with E. Buckwar and A. Thalhammer and connected with the talk *Computational mean-square stability analysis for linear systems of SODEs* by A. Thalhammer, which treats the interplay of different stability concepts in numerical simulation.