Symmetric and symplectic projection methods for differential equations on manifolds: the non-Abelian case

Michele Wandelt (Bergische Universität Wuppertal), Michael Günther, Michael Striebel

This work is concerned with symmetric and symplectic projection methods. The idea is based on symmetric projection schemes introduced by Hairer for ODE systems in the Abelian case R^n living on a manifold, which combine a symmetric scheme with a projection on the manifold, resulting in an overall symmetric scheme which preserves the constraint defined by the manifold.
We have generalized this scheme to projection schemes, which combine a symmetric, time-reversible and symplectic scheme (Leapfrog, for example) with a projection on the manifold described by the Hamiltonian, resulting in a scheme with the aforementioned properties which preserves the Hamiltonian exactly.
In a further step, we adapted the method to the non-Abelian case of matrix Lie groups. In this case, the projection method can be used in quantum field theories as, for example, in Lattice QCD. In these theories, expectation values of some operators have to be computed. This has to be done numerically with high computational cost, and the symmetric and symplectic projection method is promising to reduce this effort.