Adaptive space and time discretisations for Gross–Pitaevskii equations

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As a basic principle, benefits of adaptive discretisations are an improved balance between required accuracy and efficiency as well as an enhancement of the reliability of numerical computations. In this talk, the capacity of locally adaptive space and time discretisations for the numerical solution of low-dimensional nonlinear Schrödinger equations is investigated. The considered model equation is related to the time-dependent Gross–Pitaevskii equation arising in the description of Bose–Einstein condensates in dilute gases. The performance of the Fourier-pseudo spectral method constrained to uniform meshes versus the locally adaptive finite element method and of higher-order exponential operator splitting methods with variable time stepsizes is studied. Numerical experiments confirm that a local time stepsize control based on a posteriori local error estimators or embedded splitting pairs, respectively, is effective in different situations with an enhancement either in efficiency or reliability. As expected, adaptive time-splitting schemes combined with fast Fourier transform techniques are favourable regarding accuracy and efficiency when applied to Gross–Pitaevskii equations with a defocusing nonlinearity and a mildly varying regular solution. However, the numerical solution of nonlinear Schrödinger equations in the semi-classical regime becomes a demanding task. Due to the highly oscillatory and nonlinear nature of the problem, the spatial mesh size and the time increments need to be of the size of the decisive (small) parameter, especially when it is desired to capture correctly the quantitative behaviour of the wave function itself. The required high resolution in space constricts the feasibility of numerical computations for both, the Fourier pseudo-spectral and the finite element method. Nevertheless, for smaller parameter values adaptive time discretisations facilitate to determine the time stepsizes sufficiently small in order that the numerical approximation captures correctly the behaviour of the analytical solution. Further illustrations for Gross–Pitaevskii equations with a focusing nonlinearity or a sharp Gaussian as initial condition, respectively, complement the numerical study.