Simplifying AMF-schemes for Inexact Jacobians in large Stiff ODEs

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Splitting schemes of AMF (Approximate Matrix Factorization)-type for the implementation of Rosenbrock methods in the time integration of Advection Diffusion Reaction PDE systems semi-discretized in space by means of Finite Differences or Finite Volume, are considered.
The main point in the proposed Modified AMF schemes is that allows the use of inexact Jacobian matrices in the splitting without losing in the convergence order of the underlying Rosenbrock method, which is of great interest for non-linear problems. Besides the new Modified AMF schemes allow to deal in a explicit way with some part of the derivative function, such as the advection terms, without restricting the CFL number for pure advection problems, in a significant way.
Particular attention will be paid to the one stage Rosenbrock method (ROS1) of order two, which for autonomous ODEs is given by,

$$(I - \frac{\Delta t}{2} J_n)(y_{n+1} - y_n) = \Delta t F(y_n), \quad J_n := \frac{\partial F}{\partial y}(y_n).$$

A stability analysis for different splitting of the Jacobian and several Modified AMF approaches will be presented and some connections with existing schemes will be shown.
A 2D-Radiation-Diffusion model of very practical interest in Physics [3, 1, 2] (of parabolic type) consisting of two strongly coupled non-linear PDEs having a stiff reaction part, will be integrated with ROS1 in the different versions of the proposed AMF-schemes and with other classical methods currently used in the literature. It will be shown that the Modified AMF-iteration implemented in ROS1 supposes a noticeable improvement regarding the standard AMF implementation and also is an attractive alternative to existing integrators.

References

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