Higher order splitting methods for a class of Hamiltonian equations

Asif Mushtaq (NTNU, Trondheim), Anne Kværnø, Kåre Olaussen

A systematic procedure for increasing the accuracy of numerical solutions of a rather general class of Hamilton equations of the form

$$H(\mathbf{q},\mathbf{p}) = \frac{1}{2}\mathbf{p}^T M\mathbf{p} + V(\mathbf{q}),$$

have been discussed in [1] (and references therein). The schemes introduced here preserve the symplectic structure. The Störmer-Verlet method is of 2nd order. By the use of generating functions it can be increased from 2nd to 8th order. We have tested various orders of the method on a simple anharmonic oscillator, with regard to the very long time behaviour. In this talk, I will give equivalent graphical representations of the schemes, and test them on a wider class of problems. The procedure is conveniently described in terms of rooted trees and B-series. In general, the basic idea is to correct the Hamiltonian used in the splitting scheme with terms of increasing orders. I will show how can we construct the improved Hamiltonian up to higher order. I will also present results of numerical simulations of selected systems.

References

[1] A. Mushtaq, A. Kværnø, and K. Olaussen, *Systematic improvement of splitting methods for the Hamilton equations*, contribution to World Congress of Engineers 2012 (London, UK., 4–6 July, 2012).