Runge-Kutta projection methods for conservative and non conservative problems

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Geometric structures (first integrals, orthogonality, Lyapunov functions, symplecticity, etc.) play an important role both in qualitative and quantitative studies of the flow of differential equations. In particular, preservation of invariants of differential systems by numerical integrators is a requirement that can be very important if the qualitative properties of the solution are to be properly reproduced. Some numerical integrators based on Runge-Kutta methods combined with projection techniques can be a simple and good option to preserve invariants. In addition to orthogonal projection, other directional projections have been proposed that lead to Runge-Kutta methods, which preserve all linear first integrals and are affine invariant. However, the selection of the best direction is an open question. In the first part of this talk the search of a proper projection direction is analysed, attempting to minimize the leading error term of the projected solution. Special attention is paid to oscillatory problems, maximizing the dispersion order. In the second part of the talk, some applications of projection techniques are presented, showing how these techniques can be useful in the numerical integration of problems possessing Lyapunov functions and even for non conservative problems. Numerical experiments showing the performance of the proposed projection methods are presented.