Stroboscopic method for wave equation

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I am interested by long time numerical integration of highly oscillatory equations. Classical theory says that in order to make a good numerical approximation of the solution, the integration step must be significantly less than one period. This leads to two problems: The time of computation and the performance of computer to integrate this type of equation over millions of periods.
In the periodic case, a strategy is introduced in [1], [2] by M.P. Calvo, P. Chartier, A. Murua and J.M. Sanz-Serna and called stroboscopic method. The idea of this method is to follow the solution along another equation called averaged equation which has two interesting properties. It doesn't oscillate and coincides with the exact solution at the stroboscopic times,i.e., every multiple of the period.
Existence of this averaged equation in the ODE case has been rigorously proved for instance by L.M. Perko in [3]. The observations of P. Chartier & al. lead to a numerical method solving highly oscillatory ODEs over long time with a numerical cost independent of the ratio between the period and the final time of observation.
Perko's proof for ODE can be adapted to partial differential equation like the wave equation or the Schrödinger equation. I will explain how to adapt this proof to the semi-linear wave equation case and show some numerical results to illustrate the benefits of this method.

References

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