Adaptive Nested Implicit Runge-Kutta Methods with Global Error Control and Their Application in Fluid Mechanics

Gennady Kulikov (Instituto Superior Técnico), P. M. Lima, M. L. Morgado

This paper deals with a special family of implicit Runge-Kutta formulas of order 4. These methods are of Gauss type; i.e., they are based on the Gauss quadrature formula of orders 4. However, the methods under discussion have only explicit internal stages that lead to cheap practical implementation. Some of the stage values calculated in a step of the numerical integration are of sufficiently high accuracy that allows for dense output of the same order as the Runge-Kutta formula used. On the other hand, the designed methods are A-stable, stiffly accurate and symmetric. Moreover, they are conjugate to a symplectic method up to order 6 at least (see [1] for more details).

All of these make the new methods attractive for solving nonstiff and stiff ordinary differential equations, including Hamiltonian and reversible problems. For adaptivity, different strategies of error estimation are discussed and examined numerically, including a cheap global error estimation based on simple summation of local error estimates that are available in any embedded Runge-Kutta pair. An automatic global error control mechanism is also presented (see [2] for more details).

Then, we solve numerically a generalization of the Cahn-Hilliard continuum model for multi-phase fluids (see [3]) where the classical Laplacian has been replaced by a degenerate one (i.e., so-called p-Laplacian). This differential problem is treated successfully by a complex technique based on a triple shooting method and the above-mentioned symmetric Runge-Kutta scheme with global error control implemented in MATLAB. Results of this numerical simulation are discussed and compared with earlier computed data.

References

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