Characterisations of symmetric general linear methods and G-symplecticity

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This is the second of two talks on the design of general linear methods $(A,\, U,\, B,\, V)$ suitable for the integration of Hamiltonian or time–reversible problems. The ultimate goal is to achieve the broad range of applicability of symplectic Runge–Kutta methods, but with less implicitness in the stage matrix A.

In this talk, we discuss how zero–stability, parasitism, time–reversal symmetry and G–symplecticity may all be analysed in terms of the linear stability function,

$$M(Z) := V + BZ(I - AZ)^{-1}U, \qquad Z = \text{diag}(z_1,\, \ldots,\, z_s) \in C^{s\times s}.$$

We also characterise time–reversal symmetry and G–symplecticity in terms of the Nyquist or transfer function,

$$N(\zeta) := A + U(\zeta I - V)^{-1}U, \qquad \zeta \in C \setminus \sigma(V),$$

and show how this leads to identifying methods with both properties.