Symmetric multistep methods for constrained Hamiltonian systems

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This talk considers the numerical solution of constrained Hamiltonian systems. These are differential-algebraic equations of index 3, for which the flow on the constrained manifold is a symplectic transformation. A method of choice for the long-time integration of such problems is the Rattle algorithm. It is symmetric, symplectic, and nearly preserves the Hamiltonian (justified by a backward error analysis), but it is only of order two and thus not efficient for high accuracy requirements.
We present symmetric linear multistep methods of arbitrarily high order, and show how they can be applied to constrained Hamiltonian systems. Their implementation and computational cost is comparable to that of the Rattle algorithm. Although the method is not symplectic, we prove that it nearly conserves over long times the Hamiltonian and quadratic first integrals such as the angular momentum in N-body problems. The proof is based on techniques related to backward error analysis. The essential ingredient is the construction of adiabatic invariants, which permits to prove that the parasitic solution components remain bounded and small over long times.