Fast weak-KAM integrators for solving Hamilton-Jacobi equations

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We consider numerical schemes for Hamilton-Jacobi equations based on a direct discretization of the Lax-Oleinik semi-group which represents the solution as a minimizer of the action over continuous curves. We prove that this method is convergent with respect to the time and space stepsizes provided the solution is Lipschitz. Moreover, we prove that the numerical scheme is a *geometric integrator* satisfying a discrete weak-KAM theorem which allows to control its long time behavior. Taking advantage of a fast algorithm for computing min-plus convolutions based on the decomposition of the function into concave and convex parts, we show that the numerical scheme can be implemented in a very efficient way.