Generalized Picard Iterations with Improved Linear Convergence Properties

Boris Faleichik (Belarusian State University), Ivan V. Bondar (Belarusian State University)

This talk is about recent advances in development of generalized Picard iterations [1] which are aimed at cheap solution of nonlinear equations systems arising during implementation of implicit Runge–Kutta (IRK) methods. Our main result is the following: the proposed iterative processes converge for all IRK methods with arbitrary stepsize $h > 0$ and all linear ODE systems $y' = Jy$ which satisfy the existence and uniqueness conditions as stated in lemma 5.2.5 from [2]. Therewith these iterations are applicable in general nonlinear case, they are "matrix-free" (but require the estimate of Jacobi matrix spectral radius), and need $O(ns)$ memory storage for implementation, where n is ODE dimension and s is the number of IRK stages. So the natural purpose of generalized Picard iterations is the solution of big stiff systems with complex spectrum, where explicit Chebyshev-like methods are inapplicable and classical Newton-like iterations are too expensive.
Further properties, such as the interesting relationship between convergence rate and ODE stiffness, and numerical results will be discussed during the talk.

[1] Faleichik B. V. Explicit implementation of collocation methods for stiff systems with complex spectrum // Journal of Numerical Analysis, Industrial and Applied Mathematics. Vol. 5, no. 1-2, 2010, pp. 49-59.

[2] Dekker, K.; Verwer, J. G., Stability of Runge-Kutta Methods for Stiff Nonlinear Differential Equations. Amsterdam-New York, North-Holland 1984.