Highly stable General Linear Methods for second order Ordinary Differential Equations

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In this talk we consider the family of General Linear Methods (GLMs) for second order ordinary differential equations (ODEs). Such methods have been introduced in [2] with the aim to provide an unifying approach for the analysis of the properties of convergence, consistency and zero-stability, by extending the results obtained in the literature for GLMs solving first order ODEs [1, 3]. Our investigation is addressed to providing the building blocks useful to analyze the linear stability properties of GLMs for second order ODEs: thus, we present the extension of the classical notions of stability matrix, stability polynomial, stability and periodicity interval, A-stability and P-stability to the family of GLMs. Special attention will be focused on the practical derivation of highly stable methods, by investigating GLMs inheriting the same stability properties of highly stable numerical methods existing in literature, e.g. Runge-Kutta-Nyström methods based on indirect collocation on Gauss-Legendre points, which are known to be P-stable: this property, in analogy to a similar feature introduced for GLMs solving first order ODEs (compare [1, 3]), is called *Runge-Kutta-Nyström stability*. The stability properties of GLMs with Runge-Kutta-Nyström stability depend on a quadratic polynomial, which results to be the same stability polynomial of the best Runge-Kutta-Nyström assumed as reference. We finally provide and discuss examples of P-stable irreducible GLMs with Runge-Kutta-Nyström stability.

References

[1] J.C. Butcher, *Numerical Methods for Ordinary Differential Equations*, 2nd Edition, John Wiley & Sons, Chichester, 2008.

[2] R. D'Ambrosio, E. Esposito, B. Paternoster *General Linear methods for* $y'' = f(y(t))$, submitted.

[3] Z. Jackiewicz, *General Linear Methods for Ordinary Differential Equations*, John Wiley & Sons, Hoboken, New Jersey, 2009.