Application of the DAE theory in investigation of quasi-stationary hydraulic circuits

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We consider a quasi-stationary model of a hydraulic circuit written in the form of the differential algebraic equation

$$\begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{D}(t) \\ \dot{P}(t) \end{pmatrix} + \begin{pmatrix} S_0 & A^\top \\ A & 0 \end{pmatrix} \begin{pmatrix} D(t) \\ P(t) \end{pmatrix} + \begin{pmatrix} S|D(t)|D(t) \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} H(t) + A_1^\top P_*(t) \\ Q(t) \end{pmatrix}^\top, \quad t \in [0, \infty) \tag{1}$$

where $\begin{pmatrix} A^\top & A_1^\top \end{pmatrix} = \overline{A}$ is the incidence matrix of the graph of the hydraulic circuit under consideration;
$D(t) = (d_1(t) \quad d_2(t) \quad \dots \quad d_r(t))^\top$ is the vector-function of flow rates;
$P(t) = (p_1(t) \quad p_2(t) \quad \dots \quad p_\mu(t))^\top$ is the vector-function of the pressure at the nodes;
$P_*(t) = (p_{1,*}(t) \quad p_{2,*}(t) \quad \dots \quad p_{\mu(t),*})^\top$ is the vector-function of the known pressure;
$R = \mathrm{diag}\{\rho_1, \rho_2, \dots, \rho_r\}$ represents the momentum parameters;
$S = \mathrm{diag}\{s_1, s_2, \dots, s_r\}$ and $S_0 = \mathrm{diag}\{s_{1,0}, s_{2,0}, \dots, s_{r,0}\}$ are the resistance parameters of the branches of the hydraulic circuit;
$|D(t)|D(t) = \big((|d_1(t)|d_1(t) \quad |d_2(t)|d_2(t) \quad \dots \quad |d_r(t)|d_r(t))\big)^\top$;
$H(t) = \big((h_1(t) \quad h_2(t) \quad \dots \quad h_r(t))\big)^\top$ and $Q(t) = (q_1(t) \quad q_2(t) \quad \dots \quad q_r(t))^\top$ represent inflows and outflows correspondingly. The system is index two.
In the talk we discuss local and global existence conditions for system (1) and propose a numerical algorithm based on the reduction of the system to the index one system.

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