Heterogeneous Multiscale Methods for Highly-Oscillatory Mechanical Systems with Solution-Dependent Frequencies

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The framework of Heterogeneous Multiscale Methods (HMM) was originally proposed for the efficient computation of multiple time-scale problems. Briefly, HMM deals with systems of differential equations whose exact dynamics can be viewed as a superposition of an underlying averaged macroscale dynamics, which it approximates, and a fast microscale dynamics driving the actual motion. HMM does so without full explicit knowledge of the macroscale forces and provides the missing data via a micro-simulation in each step.

The talk examines an application of HMM to mechanical systems with solution-dependent high frequencies. It is shown that a correct initialization of the micro-simulation depends crucially on the adiabatic invariance of the actions. This almost-invariance property also guarantees the existence of an underlying effective system, which is derived. Using the example of a stiff spring double pendulum, an HMM including RATTLE as a macro-integrator is formulated. The analysis is done using canonical transformations proposed by K. Lorenz and Ch. Lubich.