On Rosenbrock methods for singular singularly perturbed problems and their application to nearly incompressible materials

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In structural mechanics, the modeling of nearly incompressible materials makes use of a constraint that measures the change of volume and that becomes part of a mixed problem formulation as transient saddle point problem. Studying the mathematical structure of the finite element discretization, the system turns out to be a differential-algebraic equation (DAE) that can be interpreted as singular singularly perturbed problem (SSP) with respect to the bulk modulus as perturbation parameter. Though this problem is for physically meaningful parameter values of index 1, it is close to an index 3 limiting system and may become hard to solve numerically. A simple example of the problem class can be stated as follows: For some given function $\phi(t)$ we want to solve

$$\begin{aligned} \ddot{q} &= \ddot{\phi} - \lambda \\ \epsilon^2 \lambda &= q - \phi. \end{aligned}$$

This model equation in the fashion of the classical Prothero Robinson equation captures already some of the relevant phenomena and is of index 1 for $\epsilon > 0$ and of index 3 for $\epsilon = 0$.

In the nonlinear case, the numerical integration of SSPs using implicit Runge-Kutta methods suffers from step size restrictions and bad convergence of Newtons-method, in combination with order reduction phenomena [1, 2]. Rosenbrock type methods avoid the use of a Newton iteration by linearization of the nonlinear equations and thus may avoid some of the mentioned convergence problems. In the talk, we are giving an overview on different Rosenbrock methods and analyze their performance and order behavior when applied to SSPs and the special class of nearly incompressible structural dynamic systems.

References

[1] C. Lubich *Integration of Stiff Mechanical Systems by Runge-Kutta Methods*, ZAMP, 44:1022-1053 (1993)

[2] B. Simeon *Order reduction of stiff solvers at elastic multibody systems*, Applied Numerical Mathematics 28:459-475 (1998)