Operator Index Reduction in Elastodynamics

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In flexible multibody dynamics, boundary conditions of the involved bodies play a crucial rule. Therefore, we analyse the equations of elastodynamics with weakly enforced Dirichlet constraints. The use of Lagrange multipliers leads to a problem of saddle point structure,

$$\begin{aligned} (\rho\ddot{u}, v)_{L^2(\Omega)} + a(u,v) + b(v,\lambda) &= \langle f, v\rangle \qquad \text{for all } v \in [H^1(\Omega)]^2, \\ b(u,\mu) \qquad\qquad &= \langle g, \mu\rangle \qquad \text{for all } \mu \in [H^{-1/2}(\partial\Omega)]^2. \end{aligned}$$

A standard semi-discretization in space by finite elements leads to a differential algebraic equation of index 3. Performing the time-integration by Newmark's method, we achieve second order convergence for the deformation variable but no convergence for the Lagrange multiplier. In the context of coupled systems, the Lagrange multiplier equals the stress in normal direction at the boundary. Thus, an index reduction such as minimal extension is advisable. This technique provides an extended index-1 formulaton which guarantees the boundary constraints, i.e., we avoid a drift at the boundary.

In this talk, we present an index reduction technique on operator level. This procedure acts on the partial differential equation, i.e., on the continuous model. The result is an extended operator DAE with the property that a semi-discretization in space leads directly to an index-1 formulation. Furthermore, we show that the index reduction and semi-discretization steps commute.