Bulk-surface Lie splitting for parabolic problems with dynamic boundary conditions

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This talk studies a novel bulk–surface splitting method of first order for (semi-linear) parabolic partial differential equations with dynamic boundary conditions, e.g., systems of the form

$$\begin{aligned} \dot{u} - \nabla \cdot (\nabla u) &= f_\Omega(u) \qquad \text{in } \Omega, \\ \dot{u} - \beta\, \Delta_\Gamma u + n \cdot \nabla u + u &= f_\Gamma(u) \qquad \text{on } \Gamma \end{aligned}$$

with $\beta \geq 0$. In general, dynamic boundary conditions appear in applications where the momentum on the boundary should not be neglected, like in models of heat sources on the boundary.

The proposed Lie splitting scheme is based on considering the boundary conditions as a second dynamical system, which is coupled to the bulk problem. The splitting approach is combined with bulk–surface finite elements and an implicit Euler discretization of the subsystems. While other known first-order splittings schemes technically do not approximate the spatially discretized system, our proposed scheme converges to the expect one at the cost of a weak coupling condition of the form $\tau \lesssim h$. The convergence is illustrated also numerically.