Superconvergent methods inspired by the Crank-Nicolson scheme in the context of diffusion PDEs (deterministic and stochastic)

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R 3.07, R 3.28@ Fri Z1 09:00-09:50

In this talk, we present two different situations where the Crank-Nicolson method is surprisingly more accurate than one could expect and inspires the design of new efficient numerical integrators:

- **in the context of splitting methods for parabolic PDEs [3, 4]:** we show that the Strang splitting method applied to a diffusion-reaction equation with inhomogeneous general oblique boundary conditions is of order two when the diffusion equation is solved with the Crank-Nicolson method, while order reduction occurs in general if using other Runge-Kutta schemes or even the exact flow itself for the diffusion part. We prove [4] these results when the source term only depends on the space variable, an assumption which makes the splitting scheme equivalent to the Crank-Nicolson method itself applied to the whole problem.
- **in the context of ergodic parabolic stochastic PDEs [5, 1, 2]:** although the Crank-Nicolson method can sample exactly the invariant measure of ergodic stochastic differential equations in the Gaussian case, it is only A-stable and lacks the L-stability property which is desirable for a fast convergence to equilibrium. Using the idea of post-processing, we investigate how the L-stability property and the exactness for the invariant measure in the Gaussian case can be achieved simultaneously. We present such schemes applied to nonlinear ergodic problems in the context of implicit Runge-Kutta methods [5] and in the context of explicit stabilized Runge-Kutta methods [1], which can be shown to be strongly convergent [2] for a class of quasilinear parabolic stochastic PDEs, including the quasilinear stochastic heat equation with space-time white noise.

References

[1] A. Abdulle, I. Almuslimani, and G. Vilmart, *Optimal explicit stabilized integrator of weak order one for stiff and ergodic stochastic differential equations*, SIAM/ASA J. Uncertain. Quantif. **6** (2018), 937–964.

[2] A. Abdulle, C.-E. Bréhier, and G. Vilmart, *Convergence analysis of explicit stabilized integrators for parabolic semilinear stochastic PDEs*, arXiv:2102.03209, Submitted.

[3] G. Bertoli and G. Vilmart, *Strang splitting method for semilinear parabolic problems with inhomogeneous boundary conditions: a correction based on the flow of the nonlinearity*, SIAM J. Sci. Comput. **42** (2020), A1913–A1934.

[4] G. Bertoli, C. Besse, and G. Vilmart, *Superconvergence of the Strang splitting when using the Crank-Nicolson scheme for parabolic PDEs with Dirichet and oblique boundary conditions*, arXiv:2011.05178, to appear in Math. Comp. (2021), 25 pages.

[5] C.-E. Bréhier and and G. Vilmart, *High-order integrator for sampling the invariant distribution of a class of parabolic stochastic PDEs with additive space-time noise*, SIAM J. Sci. Comput **38** (2016) A2283–A2306.