Schwarz and Schur time domain decomposition for nonlinear ODE

Damien Tromeur-Dervout (Université de Lyon, Institut Camille Jordan UMR5208-CNRS-U.Lyon1)
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We developed parallel time domain decomposition methods to solve systems of linear ordinary differential equations (ODEs) based on the Aitken-Schwarz [5, 3] or primal Schur complement domain decomposition methods [2]. The methods require the transformation of the initial value problem in time defined on $]0, T]$ into a time boundary values problem. Let $f(t, y(t))$ be a function belonging to $\mathcal{C}^1(\mathbb{R}^+, \mathbb{R}^d)$ and consider the Cauchy problem for the first order ODE:

$$\left\{\dot{y} = f(t, y(t)),\ t \in]0, T],\ y(0) = \alpha \in \mathbb{R}^d. \right. \tag{1}$$

The time interval $[0, T]$ is split into p time slices $S^{(i)} = [T^+_{i-1}, T^-_i]$, with $T^+_0 = 0$ and $T^-_p = T^-$. The difficulty is to match the solutions $y_i(t)$ defined on $S^{(i)}$ at the boundaries T^+_{i-1} and T^-_i. Most of time domain decomposition methods are shooting methods [1] where the jumps $y_i(T^-_i) - y_{i+1}(T^+_i)$ are corrected by a sequential process which is propagated in the forward direction (i.e. the correction on the time slice $S^{(i-1)}$ is needed to compute the correction on time slice $S^{(i)}$). Our approach consists in breaking the sequentiality of the update of each time slice initial value. To this end, we transform the initial value problem (IVP) into a boundary values problem (BVP) leading to a second order ODE:

$$\begin{cases} \ddot{y}(t) = g(t, y(t), \dot{y}(t)) \stackrel{def}{=} \dfrac{\partial f}{\partial t}(t, y(t)) + \dot{y}(t)\dfrac{\partial f}{\partial y}(t, y(t)),\ t \in]0, T[, & \text{(2a)} \\ y(0) = \alpha, & \text{(2b)} \\ \dot{y}(T) = \beta \stackrel{def}{=} f(T, y(T)) & \text{(2c)} \end{cases}$$

Then classical domain decomposition methods apply such as the multiplicative Schwarz method with no overlapping time slices and Dirichlet-Neumann transmission conditions (T.C.) for linear system of ODE (or PDE [4]). As proved in [3] the convergence/divergence of the error at the boundaries of this Schwarz time DDM can be accelerated by the Aitken technique to the right solution when $f(t, y(t))$ is linear. Nevertheless, the difficulty in solving equation (2) is that β is not given by the original IVP. In this talk, we investigate the proposed approach for nonlinear ODE.

References

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