On the exact tangent matrices of a geometrically exact beam formulated on the special Euclidean group SE(3)

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R 1.23 Tue Z3 14:00-14:30

This work presents the exact tangent matrices of a geometrically exact beam finite element formulated on the special Euclidean group SE(3)[1]. The objective is to obtain optimal convergence of Newton-Raphson iterations with an implicit time integration procedure and allow the semi-analytic calculation of the sensitivity of flexible multibody systems for optimization purposes.
A beam is a structural element having one of its dimensions much larger than the other two. The neutral axis corresponds to the beam's centerline, defined along that longer dimension and its cross-sectional plane, which is normal to the long axis. At any point of the centerline, the cross-section position and orientation are represented by a local frame. Each local frame has 6 degrees of freedom, namely three translations and three rotations. The transformation between the inertial and the local frame is represented by an element on the special Euclidean group SE (3). The beam is represented on the special Euclidean group SE(3) using a motion approach; hence, the position and rotation fields are treated as a unit. The geometric description of the element is based on the representation of the frame transformation as 4×4 homogeneous transformation matrices $\mathbf{H}$.

$$\mathbf{H} = \begin{bmatrix} \mathbf{R} & \mathbf{x} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}$$

where $\mathbf{x}$ is a 3×1 vector defining the position of the local frame and $\mathbf{R}$ is a 3×3 rotation matrix defining the orientation of the local frame. In order to express the local frames along the beam's reference curve, a two-node element is used. The finite element interpolation of the local frame is based on the exponential map of the special Euclidean group SE(3), a non-commutative and non-linear space. This finite element discretization procedure generates helicoidal shape functions.
The coupling between rotation and translation introduced by these the helicoidal functions yields a naturally locking-free element that can efficiently capture non-linear deformation patterns of elastic beams. For instance, when applied to a pure bending/torsion case, a single element can be used and

the exact solution achieved. The equations of motion generated from this approach take the form of second-order differential-algebraic equations on a Lie group; they are solved using a Lie group time integration scheme, namely, the generalized-α method proposed by Brüls et al [2].

The non-linear set of equations and the iterative Newton-Raphson time integration require that the tangent matrices must be computed at each iteration. Indeed, the more accurate the tangent matrices are calculated, the less is the possibility of divergence and the number of iterations needed to reach the desired numerical tolerance. It has an advantage when extremely bent/twisted beams need to be captured while employing relatively coarse meshes and large time steps. Therefore, in order to achieve higher accuracy, the exact tangent matrices, in other words, all the terms in the linearized equation of motion, have to be considered; these terms were entirely derived herein. This has been hardly addressed in the literature. In addition, exact tangent matrices are mandatory for semi-analytic calculation of the sensitivity analysis of both the adjoint variable and direct differentiation methods[3].

Moreover, numerical and implementation issues related to truncation errors, the range of applicability and singularity were addressed and explored. The exponential map and its tangent operator have two options to be calculated. The first option is an infinite serial expansion; hence, it must be truncated to a finite number of terms. Consequently, it has a truncation error. However, the truncation error decreases as it approaches the origin of the exponential map of the SE(3). The second option is the serial expansion‘s close-form which is an expression composed of trigonometric terms. However, it suffers from a singularity at the origin. Thus, the error increases approaching the origin. Regarding the tangent matrices, it uses the tangent operator and its directional derivatives. In previous studies, the directional derivatives have been essentially computed using trigonometric close-form. To circumvent singularity and accuracy issues typical of the trigonometric approach, we proposed an approach based on an expansion series; this has the advantage of high precision close to the origin, and it is free of singularity at the origin. Two numerical tests were performed to assess the approach developed here. The first test showed a relative error of the tangent operator and its directional derivatives considering both the series and close-form expressions. Therefore, it allowed the development of a strategy to control the numerical error according to the desired tolerance. The results indicated that a switch between the series and close-form expressions for the tangent operator and its directional derivatives in the range $[0, \pi]$ is recommended to guarantee high accuracy and cost-effective computations. The second numerical test assured that the exact tangent matrix was obtained for the geometrically exact beam formulated on the special Euclidean group SE(3). The error analyses were compared against the finite-difference method, and the results indicated that the exact matrix was obtained by using the methods developed here.

The exact tangent matrices developed here, the numerical strategies and the tests performed have attested that an optimal convergence can be obtained using the iterative Newton-Raphson method. The approaches developed were further explored using detailed sensitivity analyses. The methods developed here proved to be robust, cost-effective and appropriate to be applied on flexible multibody systems where optimization techniques are crucial.

[1] Sonneville, V., Cardona, A. and Brüls, O., Geometrically exact beam finite element formulated on the special Euclidean group SE (3). Computer Methods in Applied Mechanics and Engineering 268(2014): 451-474.

[2] Sonneville, V. and Brüls, O., Sensitivity analysis for multibody systems formulated on a Lie group. Multibody System Dynamics 31, no. 1 (2014): 47-67.

[3] Brüls, O., Cardona, A., Arnold, M.: Lie group generalized-α time integration of constrained flexible multibody systems. Mech. Mach. Theory 48,(2012): 121–137