Continuous-time extensions of stochastic one-step methods

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In this work we focus our attention on the development of continuous extensions to stochastic one-step methods for the time discretization of Stochastic Differential Equations (SDEs) [1, 2]

$$X(t) = X(t_0) + \int_{t_0}^{t} f(X(s))ds + \int_{t_0}^{t} g(X(s))dW(s), \quad t \in [t_0, T], \qquad (1)$$

where $W(t)$ is a multidimensional standard Wiener process. Inspired by the idea of deterministic numerical collocation [5, 6], we provide a continuous time extension of stochastic one-step methods, by imposing that the solution of (1) can be approximated with a piecewise linear polynomial. A dense numerics output allows to provide a more efficient error estimate and it is a very effective for a variable step-size implementation [4]. We show the constructive technique and provide selected numerical experiments confirming the effectiveness of the proposed approach.

References

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