Long-term analysis of stochastic Hamiltonian systems under time discretizations

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In this talk, we focus our investigation on providing long-term estimates of the Hamiltonian deviation computed along numerical approximations to the solutions of stochastic Hamiltonian systems, both of Itô and Statonovich types. It is well-known that the expected Hamiltonian of an Itô Hamiltonian system with additive noise exhibits a linear drift in time [2], while the Hamiltonian function is conserved along the exact flow of a Stratonovich Hamiltonian system [3, 4]. Here, we address our attention to providing modified differential equations associated to suitable discretizations for above problems, by means of weak backward error analysis arguments [1, 5, 6]. Then, long-term estimates are provided both for Itô and Stratonovich Hamiltonian systems, revealing the presence of parasitic terms affecting the overall conservation accuracy. Finally, selected numerical experiments are provided to confirm the theoretical analysis. This talk is based on a joint work with Raffaele D'Ambrosio (University of L'Aquila).

References

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