Higher order phase averaging for highly oscillatory systems

Werner Bauer @ (Imperial College London), Colin J. Cotter, Beth Wingate

R 3.07 Thu Z1 16:00-16:30

We introduce a higher order phase averaging method for nonlinear oscillatory systems. Phase averaging is a technique to filter fast motions from the dynamics whilst still accounting for their effect on the slow dynamics. Phase averaging is useful for deriving reduced models that can be solved numerically with more efficiency, since larger timesteps can be taken. Recently, Haut and Wingate (2014) introduced the idea of computing finite window numerical phase averages in parallel as the basis for a coarse propagator for a parallel-in-time algorithm. In this contribution, we provide a framework for higher order phase averages that aims to better approximate the unaveraged system whilst still filtering fast motions. Whilst the basic phase average assumes that the solution is independent of changes of phase, the higher order method expands the phase dependency in a basis which the equations are projected onto. We illustrate the properties of this method on an ODE that describes the dynamics of a swinging spring due to Lynch (2002). Although idealized, this model shows an interesting analogy to geophysical flows as it exhibits a slow dynamics that arises through the resonance between fast oscillations. On this example, we show convergence to the non-averaged (exact) solution with increasing approximation order also for finite averaging windows. At zeroth order, our method coincides with a standard phase average, but at higher order it is more accurate in the sense that solutions of the phase averaged model track the solutions of the unaveraged equations more accurately.