Numerical Treatment for an Isogeometric One-Dimensional Model for Developable Flexible Elastic Strips

Benjamin Bauer (Fraunhofer Institute for Industrial Mathematics (ITWM)), Michael Roller, Joachim Linn, Bernd Simeon

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Numerous engineering applications deal with thin-walled structural parts, an example being flexible flat cables in the development of consumer electronics or computer hardware. Classical shell models describe such slender objects at hand of their centre-surface in order to reduce the involved number of degrees of freedom and thereby the numerical costs.
Several research contributions [1] within the last century continued this idea of dimensional reduction for isometric deformations of developable base surfaces. Recently, this lead to shell descriptions depending on one parameter and, therefore, resembling rod models. For example, Starostin and van der Heijden [2] based their model on the envelope of rectifying planes.
We circumvent problems arising with vanishing curvature of the centre line by utilising a relatively parallel frame [3] along the base curve. This enables us to generalise the concept of rectifying developable surfaces to curves with much softer regularity requirements.
Isometric deformations of the centre surface preserve the developabilty of a plane reference configuration. Thus, there is no membrane strain involved and the stored energy functional consists only of the bending energy. An optimisation problem with highly non-linear geometric constraints and boundary conditions yields the equilibrium state as a local minimum.
We discuss numerical issues associated with the applied penalty formulation and isogeometric discretisation at hand of use cases for strips clamped at both ends.

References

[1] R. Fosdick, E. Fried: The Mechanics of Ribbons and Möbius Bands. Springer, Netherlands (2016)

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[3] R.L. Bishop: There is More than One Way to Frame a Curve. Am. Math. Mon. 82(3), 246–251 (1975)