Boundary value methods for semi-stable differential equations

Paul Andries Zegeling (Utrecht University)

In this talk I present a boundary-value method (BVM) that can be used for partial (PDE) and ordinary differential equation (ODE) models with semi-stable, or even ill-posed, properties. Traditionally, step-by-step methods, such as Runge-Kutta and linear multistep methods, are being utilized for time-dependent models. However, their numerical stability regions (this holds both for explicit and implicit methods) are usually such that a significant part does not intersect with areas in the complex plane which are of importance for a successful time-integration. BVMs, that need an extra numerical condition at the final time, are global methods and are, in some sense, free of such barriers. As an example, a BVM, based on the explicit midpoint method combined with an implicit-Euler final condition, possesses the whole complex plane (excluding the imaginary axis) as stability region. On the other hand, they loose efficiency, since an extended linear or nonlinear system has to be solved for the whole time range of interest. Numerical experiments illustrating these properties are given for, among others, a dispersive wave equation and the backward heat equation.