Singular value decay of solutions to operator-valued differential Lyapunov and Riccati equations

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It is frequently observed in practice that the singular values of the solutions to differential Lyapunov equations or differential Riccati equations decay very quickly. This is the basis for the low-rank approach which is often used in numerical methods for such equations: if a fast decay is not present, the solution approximant is either no longer of low rank or no longer a good approximant. In the former case, the computational cost and memory requirements become infeasible, and in the latter case the result is worthless. In spite of this, the literature contains very few, or any, theoretical results on when such decay is to be expected. The situation is better understood for algebraic Lyapunov and Riccati equations, but these results are not directly applicable to the differential case.

In this talk I will discuss recent results on extending the algebraic results to the differential case. The main result is that one should not expect exponential decay, but exponential in the negative square root. We consider the operator-valued setting, with the standard assumption that the state operator A generates an analytic semigroup and the input and output operators B and C are not too unbounded. This corresponds, e.g., to the control of abstract parabolic problems where the control may act either in a distributed fashion or through the boundary conditions. In the commonly considered matrix-valued case, which corresponds to a spatial discretization of the operator-valued equation, exponential decay has been demonstrated. I will show by an example that this is only relevant for small-scale problems; as the discretization is refined this decay deteriorates and becomes exponential in the negative square-root.