Multilevel Uncertainty Quantification with Sample-Adaptive Model Hierarchies

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Sample-based multilevel uncertainty quantification tools, such as multilevel Monte Carlo, multilevel quasi-Monte Carlo or multilevel stochastic collocation, have recently gained huge popularity due to their potential to efficiently compute robust estimates of quantities of interest (QoI) derived from PDE models that are subject to uncertainties in the input data (coefficients, boundary conditions, geometry, etc). Especially for problems with low regularity, they are asymptotically optimal in that they can provide statistics about such QoIs at (asymptotically) the same cost as it takes to compute one sample to the target accuracy. However, when the data uncertainty is localised at random locations, such as for manufacturing defects in composite materials, the cost per sample can be reduced significantly by adapting the spatial discretisation individually for each sample. Moreover, the adaptive process typically produces coarser approximations that can be used directly for the multilevel uncertainty quantification. In this talk, we present two novel developments that aim to exploit these ideas. In the first part we will present Continuous Level Monte Carlo (CLMC), a generalisation of multilevel Monte Carlo (MLMC) to a continuous framework where the level parameter is a continuous variable. This provides a natural framework to use sample-wise adaptive refinement strategies, with a goal-oriented error estimator as our new level parameter. We introduce a practical CLMC estimator (and algorithm) and prove a complexity theorem showing the same rate of complexity as MLMC. Also, we show that it is possible to make the CLMC estimator unbiased with respect to the true quantity of interest. Finally, we provide two numerical experiments which test the CLMC framework alongside a sample-wise adaptive refinement strategy, showing clear gains over a standard MLMC approach with uniform grid hierarchies. In the second part, we extend the sample-adaptive strategy to multilevel stochastic collocation (MLSC) methods providing a complexity estimate and numerical experiments for a MLSC method that is fully adaptive in the dimension, in the polynomial degrees and in the spatial discretisation.

This is joint work with Gianluca Detommaso (Bath), Tim Dodwell (Exeter) and Jens Lang (Darmstadt).