Construction of Strong Stability Preserving Implicit-Explicit General Linear Methods

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Many practical problems in science and engineering are modeled by large systems of ordinary differential equations (ODEs) which arise from space discretization of partial differential equations (PDEs). For such differential systems there are often natural splittings of the right hand sides into two parts, so they can be written in the form

$$\begin{cases} y'(t) = f\big(y(t)\big) + g\big(y(t)\big), & t \in [t_0, T], \\ y(t_0) = y_0, \end{cases}$$

$y_0 \in \mathbb{R}^m$, $f : \mathbb{R}^m \to \mathbb{R}^m$, $g : \mathbb{R}^m \to \mathbb{R}^m$, where $f(y)$ represents the non-stiff processes, for example advection, and $g(y)$ represents stiff processes, for example diffusion or chemical reaction, in semidiscretization of advection-diffusion-reaction equations. For efficient integration of such kind of systems we consider the class of implicit-explicit (IMEX) methods, where the non-stiff part $f(y)$ is integrated by an explicit numerical scheme, and the stiff part $g(y)$ is integrated by an implicit numerical scheme. After the investigation of IMEX Runge-Kutta (RK) methods [1], we consider IMEX General Linear Methods (GLMs) to obtain methods where the explicit part has the so-called strong stability preserving (SSP) property [2, 3, 4], and the implicit part of the method is A-, or L-stable. Since the good properties of the explicit and implicit part do not ensure good performances when the two schemes interact with each other, we also analyze the absolute stability of the overall IMEX method to obtain large regions of *combined stability*. We provide examples of IMEX GLMs with order $p \leq 4$ and high stage order, $q = p$, and report the results of numerical experiments based on the solution of several large stiff problems, that confirm that the proposed methods have good performances.

References

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