Some aspects of the time integration of multidimensional parabolic problems with mixed derivatives

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We start by reviewing a few schemes based on directional splitting for the time integration of multidimensional parabolic problems in case that mixed derivatives are present and where it is assumed a spatial semi-discretization based on central differences. Then, we focus on unconditional stability aspects, particularly on W-methods based on the Approximated Matrix Factorization (AMF) to perform the directional splitting. The linear constant coefficient problem with Homogeneous Boundary Conditions of Dirichlet type will be analyzed and a relevant scalar test problem stemming from it, will play a relevant role in the stability analysis. The empirical order of convergence in PDE sense of some relevant schemes will be illustrated with a few linear test PDE problems, one of them meeting applications in Finance. It will be seen that often the convergence order presents a stronger reduction when the boundary conditions are time-dependent. A way to circumvent this drawback will be shown.

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