A Semi-Discrete Numerical Method for Convolution-Type Unidirectional Wave Equations

Husnu Ata Erbay (Ozyegin University), Saadet Erbay, Albert Erkip

In this study we prove the convergence of a semi-discrete numerical method applied to the initial value problem for a general class of nonlocal nonlinear unidirectional wave equations $u_t + (\beta * f(u))_x = 0$. Here the symbol * denotes the convolution operation in space, $(\beta * v)(x) = \int_{\mathbb{R}} \beta(x-y)v(y)dy$, and the kernel β is even function with $\int_{\mathbb{R}} \beta(x)dx = 1$. Members of the class arise as mathematical models for the propagation of dispersive waves in a variety of situations. For instance, the Benjamin-Bona-Mahony equation and the Rosenau equation are members of the class. Our calculations closely follow the approach in [1] where error analysis of a similar semi-discrete method was conducted for the nonlocal bidirectional wave equations. As in [1], the numerical method is built on the discrete convolution operator based on a uniform spatial discretization. The semi-discretization in space and a truncation of the infinite spatial domain to a finite one give rise to a finite system of ordinary differential equations in time. We prove that solutions of the truncated problem converge uniformly to those of the continuous one with the second-order accuracy in space when the truncated domain is sufficiently large. Finally, for some particular choices of the convolution kernel, we provide numerical experiments that corroborate the theoretical results.

[1] H.A. Erbay, S. Erbay and A. Erkip, Convergence of a semi-discrete numerical method for a class of nonlocal nonlinear wave equations, arXiv:1805.07264v1 [math.NA] (to be published in ESAIM: Mathematical Modelling and Numerical Analysis).