Stability issues in the discretization of stochastic differential equations

Raffaele D'Ambrosio (University of L'Aquila)

The aim of this talk is the analysis of various stability issues for numerical methods designed to solve stochastic differential equations. We first aim to consider nonlinear Itô stochastic differential equations (SDE): under suitable regularity conditions, exponential mean-square stability holds, i.e. any two solutions $X(t)$ and $Y(t)$ of a SDE with $\mathbb{E}|X_0|^2 < \infty$ and $\mathbb{E}|Y_0|^2 < \infty$ satisfy

$$\mathbb{E}|X(t) - Y(t)|^2 \leq \mathbb{E}|X_0 - Y_0|^2 e^{\alpha t}, \tag{1}$$

with $\alpha < 0$. We aim to investigate the numerical counterpart of (1) when trajectories are generated by stochastic linear multistep methods, in order to provide stepsize restrictions ensuring analogous exponential mean-square stability properties also numerically [1, 4]. This is a joint research with Evelyn Buckwar (Johannes Kepler University of Linz).

We next consider second order stochastic differential equations describing the position of a particle subject to the deterministic forcing $f(x)$ and a random forcing $\xi(t)$ of amplitude ε. The dynamics exhibits damped oscillations, with damping parameter η. We aim to analyze long-term properties for indirect stochastic two-step methods, with special emphasis to understanding the ability of such methods in retaining long-term invariance laws [2, 3]. This is a joint research with Martina Moccaldi and Beatrice Paternoster (University of Salerno).

References

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