Forward option pricing using Gaussian RBFs

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We will present a method to numerically price options by solving the Fokker-Planck equation. The solution to this partial differential equation (PDE) describes the evolution of the conditional probability density $p(s,t)$ for the value s of the underlying asset at time t, given that the value is s_0 at time t_0. This enables the pricing of several contracts with pay-offs $\phi(s,K,T)$ (with strike-price K and time of maturity T) by simply integrating the conditional probability density function at time of maturity with the pay-off function for each contract.
This means that our method only requires the solution of one PDE to price several contracts. This is useful in practical applications where it is common to price many contracts simultaneously for the same underlying diffusion model.
From a numerical perspective the initial condition for the Fokker-Planck equation $p(s_0,t_0)$ is particularly challenging since it is a Dirac delta function. In [1] a closed-form expansion for the conditional probability density was introduced that is valid for small time-steps. We use this for the computation of $p(s,\Delta t)$ the first time-step. For the remaining time-steps we discretize the Fokker-Planck equation using BDF-2 in time and Radial Basis Function (RBF) approximation in space with Gaussian basis functions.
We will demonstrate the good qualities of our proposed method for European call options and barrier options.

References

[1] Y. Aït-Sahalia, *Maximum-likelihood estimation of discretely-sampled diffusions: A closed-form approximation approach*, Econometrica, 70: 223–262, 2002.