

The backward error of the Leja method

Peter Kandolf (University of Innsbruck), Marco Caliari, Alexander Ostermann, Stefan Rainer

The Leja method is a well established scheme for computing the action of the matrix exponential, see [2]. We present a new backward error analysis, see [3], that allows for a more efficient method. From a scalar computation in high precision we predict the necessary number of scaling steps based only on a rough estimate of the field of values or norm of the matrix and the desired backward error. We use the convergence behaviour of the scalar case on ellipses in the complex plane to get a bound for the matrix argument.

We focus on the development of the error estimation and present some numerical experiments to illustrate its behaviour. In comparisons focusing on the amount of matrix-vector products needed for the interpolation we can show that for a wide class of matrices the Leja method saves matrix-vector products in comparison to the Taylor method, see [1].

References

- [1] Al-Mohy, A.H., Higham, N.J., 2011. Computing the action of the matrix exponential, with an application to exponential integrators. *SIAM J. Sci. Comput.* 33 (2), 488-511.
- [2] Caliari, M., Kandolf, P., Ostermann, A., Rainer, S., 2014. Comparison of software for computing the action of the matrix exponential. *BIT Numerical Mathematics*, 54 (1), 113-128.
- [3] Caliari, M., Kandolf, P., Ostermann, A., Rainer, S., 2014. A backward error analysis for the Leja method. preprint, University of Innsbruck.